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Engineering Application of Time-changed Lévy Process to Capture Jumps in Stock Market

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Abstract

In financial engineering, volatility in the stock return processes is one of crucial factors when we deal with asset pricing and risk management. Besides the continuous part and the big jumps, there are a great amount of small jumps in stock prices. In this paper, under the continuous-time financial framework, we use the time-changed Lévy process with infinite activity and infinite variation to construct the SVNIG model, which can capture small jumps. This model can describe the continuous volatility component and the jump component simultaneously. MCMC approach is then employed to estimate parameters and identify latent variables. Using Hushen300 composite index in China and Hang Seng index in Hong Kong, the empirical results show that there are massive small jumps in both markets, and SVNIG model can describe jump behavior more accurately than other models.

Keywords: Jump behavior, Lévy process, MCMC; financial engineering

1. Introduction

In most of the time, the change of stock prices is moderate. However, sometimes the change is huge: stock prices jump upward or downward abruptly. In financial engineering literature, this phenomenon is called jump behavior. The happening of big jumps is rare, but once it happened, the impact on stock market, bonds market and derivatives market is tremendous. In history, most of the financial crisis began from big downward jumps, such as the1987 stock market crash in US.

Big jumps in stock markets are rare event. Since Poisson process can describe rare event perfectly, it was introduced to financial engineering domain for asset pricing. Press (1967) was the first that put forward compound event model, in which the Brownian motion is used to describe continuous movement and Poisson process is use to describe jump behavior. Using daily data, Beckers (1981), Ball and Torous (1993), Das and Sundaram (1999) all showed the existence of jumps.

In the past few decades, financial engineering relied heavily on Brownian motion and Poisson process as modeling building blocks. However, they are only two special cases of a much more general class of stochastic processes, namely Lévy process. Lévy processes have become increasingly popular in recent years, and various Lévy models have been developed in financial engineering, as Lévy process can explain quite a few new financial phenomenon. Li (2009) showed that small jumps are bigger than Brownian motion, but smaller than big jumps.

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Huang and Wu (2004) showed that those high-frequency small jumps cannot be described by jump-diffusion process. Actually, Lévy process is one good tool when describing small jumps: it has the ability to capture big and small jumps simultaneously. Carr and Wu (2004) found VG (Variance Gamma) could capture all kinds of jumps. Todorov and Tauchen (2011) showed that some indexes can be described perfectly only by Lévy jumps.

Although these Lévy processes based models have more power in describing stock price process, it is difficult to estimate. For some Lévy processes, they don't possess some moment conditions, which make us difficult to use GMM; for other Lévy process, they don't have probability density function, which make us difficult to use MLE. In this paper, we use MCMC method to estimate the parameters and latent variables.

The remaining of the paper is organized as follows. Section 2 describes and derives the Lévy process; Section 3 reviews model setting; and Section 4 describes the data and the empirical results; and finally Section 6 concludes the paper.

2. Time-changed Lévy process

2.1 Lévy process

Under a given probability space (Ω, F, P) and the complete filtration $(F_t)_{t\geq 0}$, a Lévy process X_t is the cádlág stochastic process with the independent and stationary increments and $X_0 = 0$. By the Lévy-Kintchine theorem, the characteristic function of X_t has the following form, $\psi_X(u)$ is characteristic exponential and its expression is

$$\varphi_X(u) = i\mu u - \frac{1}{2}\sigma^2 u^2 + \int_{R_0} e^{iux-1-iux1_{|x|<1}}\pi(dx)$$

Here $(\mu,\sigma,\pi(dx))$ is called Lévy triplet or characteristic triplet with $u \in R, \mu \in R, \sigma \in R_+$. Every Lévy process has one corresponding unique characteristic triplet. $\pi(dx)$ Is defined as Lévy measure on $R_0 = R\setminus\{0\}$, which is used to measure the characteristics of jump component in Lévy process. Any jump component could be described by the corresponding Lévy measure, which can control jump size and jump intensity simultaneously. For Lévy-Kintchine theorem, the more detailed proof can refer to Sato (1999), Cont and Tankov (2004). If Lévy measure has the following properties,

$$\int_{R_0} \pi(dx) = \infty \ ,$$

The Lévy process can generate infinite jump behaviours in one unit time. And hence, this kind of Lévy processes is called infinite activity Lévy process. On the contrary, if

$$\int_{R_0} \pi(dx) < \infty,$$

We call it finite activity Lévy process, which can only generate finite jump behaviour in one unit time. For the infinite activity Lévy process, if it satisfy

$$\int_{R_0} (|x| \wedge 1)\pi(x)dx = \infty,$$

then we call it infinite variation; on the contrary, if

$$\int_{R_0} (|x| \wedge 1)\pi(x)dx < \infty,$$

we call it finite variation. In both cases, $(|x| \wedge 1)$ is truncated function which behalf the minimum of 1 and $|x|$ $B(t;\theta,\sigma)$ is one Brownian motion with drift term, and the drift parameter is θ and the diffusion standard deviation is σ, the concrete form is:

$$B(t;\theta,\sigma) = \theta t + \sigma\sqrt{t}W_t,$$

where W_t is one stochastic variable with standard normal distribution. Assume the time t function $G(t;1,v)$ follows Γ distribution with mean μ and variance v, here we set $\mu = 1$, then

$$X_t = B(G(t;1,v);\theta,\sigma_J),$$
$$=\gamma G(t;1,v)+\sigma_J\sqrt{G(t;1,v)}W_t,$$

follows Variance Gamma (VG) process; Using the same procedure, if we set $G(t;1,v)$ to follow inverse Gaussian distribution, then X_t follows Normal Inverse Gaussian (NIG) process. Both VG and NIG process have infinite activity properties. But NIG process has infinite variation and VG with finite variation. Both of them belong to infinite Lévy process, the corresponding characteristic exponents are

$$\begin{cases} \phi_{VG}(u) = -\frac{1}{k}\log(1+\frac{u^2\sigma^2 k}{2} - i\theta k u), \\ \phi_{NIG}(u) = \frac{1}{k} - \frac{1}{k}\sqrt{1+u^2\sigma^2 k - 2i\theta u k} \end{cases}$$

Based on the above two formulas, we know VG and NIG are pure jump process with no continuous component.

2.2 Using stopping time to describe continuous volatility

In the above Lévy process, parameter σ is used to describe continuous volatility and it is constant. This kind of assumption does not come along with the observation and can be resolved by constructing stopping time. Consider to define one cádlág and positive stochastic process V_t in the probability space $(\Omega, F, (F_t)_{t\geq 0}, P)$, we use it to construct stopping time T_t,

$$T_t = \int_0^t V_s d_s, \ t \leq T$$

Here T_t is bounded, and we call it subordinator, where V_t is instantaneous volatility. Intuitively, t is calendar time and T_t is the business time. A more active business day, captured by a higher activity rate, generates higher volatility for asset returns. The randomness in business activity generates randomness in volatility.

In the empirical analysis, V_t is assumed to follow the CIR process with positive value, mean-reverting properties. These properties make CIR appropriate to describe volatility (Carr and Wu, 2004). The representation of CIR process is

$$dV_t = \theta(\eta - V_t)d_t + \sigma\sqrt{V_t}dW_t,$$

where W_t is one standard Brownian motion. Suppose X_t is one Lévy process and use stopping time T_t to construct a new Lévy process which could be represented by:

$$X_t = X_{T_t} = X(T_t),$$

Carr and Wu (2003, 2004) used time-changed Lévy process to capture different component in volatility process which could capture volatility clustering. Bates (2005) used time-changed Lévy process to analyse the ways by which return and volatility influence each other.

3. Model setting

3.1 Continuous-time model

Assume $(\Omega, F, (F_t)_{t\geq 0}, P)$ is one complete stochastic space, the price process can be described by drift item, diffusion item and jump component. The jump component can be captured by generalized Lévy process, which can be fininte or infinite variation. Based on this, we construct two different model, one is SVVG mode, which is infinite activity with finite variation; the other is SVNIG model, which is infinite activity with infinite variation.

Model 1: SVVG

$$\begin{cases} S_t = S_0 \exp(\mu t + \sqrt{V_t}W_t + J_t^c), \\ dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dB_t, \\ J_t^c = \gamma G_t^c + \sigma\sqrt{G_t^c}dZ_t, \\ G_t^c \sim Gam(t; 1, \nu), \\ [dW_t, dB_t] = \rho dt \end{cases}$$

where μ is drift item; W_t and B_t are both standard Brownian motion, if they are correlated, the model exists leverage effect; V_t is instantaneous variance, which used to describe the continuous volatility part in price process; variable J_t^c is the cumulative jump component at time t, which follows VG process; G_t^c is the latent variable; ε_t follows standard normal distribution, $\varepsilon_t \sim N(0,1)$, and it is independent with respect to W_t and B_t In the model, the parametric set can be represented by $\Theta = \{\mu, \kappa, \theta, \rho, \sigma_V, \gamma, \sigma, \upsilon\}$, and there are three latent variables, $\{V_t\}_{t=1}^T$, $\{J_t^c\}_{t=1}^T$, $\{G_t^c\}_{t=1}^T$. The construction of the model is just to add VG-type jumps into SV model, so we call it SVVG model.

Model 2: SVNIG

Keeping the other assumptions unchanged, only suppose variable G_t which is used to construct time-changed Brownian motion to follow the Inverse Gaussian distribution, then we obtain the SVNIG model as follows,

$$\begin{cases} S_t = S_0 \exp(\mu t + \sqrt{V_t} W_t + J_t^c), \\ dV_t = \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t} dB_t, \\ J_t^c = \gamma G_t^c + \sigma \sqrt{G_t^c} dZ_t, \\ G_t^c \sim\sim IG(t;1,v), \\ [dW_t, dB_t] = \rho dt \end{cases}$$

In model 2 the parameter set is $\Theta = \{\mu, \kappa, \theta, \rho, \sigma_v, \gamma, \sigma, \upsilon\}$, and three latent variable series are $\{V_t\}_{t=1}^T$, $\{J_t^c\}_{t=1}^T$ and $\{G_t^c\}_{t=1}^T$ respectively.

3.2 State-space model

To estimate the two models conveniently, we must discretize the two models. Suppose $y_t = \ln S_t$, where $t = 1, \quad, T$. The price process is continuous but we can only observe it in fixed time interval. Here we adopt the Euler approach, and we obtain the following state-space models.

SVVG's state-space model

$$\begin{cases} y_{t+1} = y_t + \mu\Delta + \sqrt{V_t \Delta}\varepsilon_{t+1}^y + J_{t+1}, \\ V_{t+1} = V_t + \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t \Delta}\varepsilon_{t+1}^v, \\ J_{t+1} = \gamma G_{t+1} + \sigma_J G_{t+1} \varepsilon_{t+1}^J, \\ G_{t+1} \sim \Gamma(\Delta, v), \\ corr(\varepsilon_{t+1}^y, \varepsilon_{t+1}^v) = \rho \end{cases}$$

In the model, ϵ_{t+1}^y, ϵ_{t+1}^v and ϵ_{t+1}^J follow standard normal distribution, in which ϵ_{t+1}^y and ϵ_{t+1}^v are correlated with coefficient ρ; ϵ_{t+1}^J is independent with the other two noise process; where the discrete variable J_{t+1} is obtained by the formula $J_{t+1} = J_{t+\Delta}^c - J_t^c$, and $G_{t+1} = G_{t+\Delta}^c - G_t^c$. Actually, this model is studied by Li et. al (2008).

SVNIG's state-space model

$$\begin{cases} y_{t+1} = y_t + \mu\Delta + \sqrt{V_t \Delta}\varepsilon_{t+1}^y + J_{t+1}, \\ V_{t+1} = V_t + \kappa(\theta - V_t)dt + \sigma_v \sqrt{V_t \Delta}\varepsilon_{t+1}^v, \\ J_{t+1} = \gamma G_{t+1} + \sigma_J G_{t+1} \varepsilon_{t+1}^J, \\ G_{t+1} \sim IG(\Delta, v), \\ corr(\varepsilon_{t+1}^y, \varepsilon_{t+1}^v) = \rho \end{cases}$$

The variables in this model have the same meaning corresponding to the previous one. We use MATLAB software to estimate the two models, so the arrangement of the data complied with the software. The observable variable is log price series $\{y_t\}_{t=1}^T$, and the latent variable series are $X = \{\{V_t\}_{t=1}^T, \{J_t\}_{t=2}^T, \{G_t\}_{t=2}^T\}$, parameter set is $\Theta = \{\mu, \kappa, \theta, \rho, \sigma_v, \gamma, \sigma, \upsilon\}$. Most of the parameters in the two models have standard posterior distributions, so we can use Gibbs sampling method to calculate the posterior mean and standard deviation. To the latent variable, we can use slice sampling method.

If the model setting is right, we can obtain the residuals which follow standard normal distribution,

$$\varepsilon_{t+1}^y = \frac{y_{t+1} - y_t - \mu\Delta - J_{t+1}}{\sqrt{V_t \Delta}} \sim N(0,1)$$

Naturally, we can use Kolmogorov-Smirnov test to check that.

3.3 Econometric estimation

In this section, we discuss how to use MCMC approach to estimate the two different models. This method use observable data and latent variable at every time point t to obtain posterior mean and standard deviation. For example, we use observable data $Y = \{y_t\}_{t=1}^T$, parameter set Θ and latent variable X to get $p(X, \Theta \mid Y)$. Using Bayes rule, posterior distribution is proportional to likelihood function times prior distribution,

$$p(X, \Theta \mid Y) \propto p(Y \mid X, \Theta) p(X \mid \Theta) p(\Theta),$$

where $p(Y \mid X, \Theta)$ is the likelihood function given parameter set and latent variable; $p(X \mid \Theta)$ is latent variable's distribution function given parameter Θ; $p(\Theta)$ is the prior distribution of parameter Θ

In most cases, direct sampling from the posterior distribution $p(X,\Theta \mid Y)$ is impossible because of its high dimension and complicated form. Based on C-H theorem (Hammersley and Cliord,1970; Besag,1974), we can then iteratively draw from its full conditionals $p(\Theta \mid X,Y)$ and $p(X \mid \Theta,Y)$, using Gibbs sampling method. Based on Bayes rule, we can obtain the joint posterior distribution by using observable log stock prices and latent variables,

$$
\begin{aligned}
p(\Theta,V,J,G \mid Y) &\propto p(Y,\Theta,V,J,G) = P(Y,V \mid J)p(J \mid G,\Theta)p(G \mid \Theta)p(\Theta) \\
&\propto \prod_{t=1}^{T-1} \frac{1}{\sigma_v V_t \Delta\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left((\varepsilon_{t+1}^y)^2 - 2\rho\varepsilon_{t+1}^y\varepsilon_{t+1}^v + (\varepsilon_{t+1}^v)^2\right)\right) \\
&\times \prod_{t=1}^{T-1} \frac{1}{\sigma_J\sqrt{G_{t+1}}} \exp\left(-\frac{(J_{t+1}-\gamma G_{t+1})^2}{2\sigma_J^2 G_{t+1}}\right) \\
&\times \prod_{t=1}^{T-1} \frac{\Delta}{\sqrt{v}} G_{t+1}^{3/2} \exp\left(-\frac{(G_{t+1}-\Delta)^2}{2vG_{t+1}}\right) \\
&\times p(\Theta),
\end{aligned}
$$

where $\epsilon_{t+1}^y = (y_{t+1} - y_t - \mu\Delta - J_{t+1}) / \sqrt{V_t\Delta}$, $\epsilon_{t+1}^y = (V_{t+1} - V_t - \kappa(\theta - V_t)\Delta) / (\sigma_v\sqrt{V_t\Delta})$. After that, we can derive the concrete posterior distribution for every parameter.

3.4 Parameter sampling

In the model, most of the parameters have standard posterior distribution, we can use Gibbs sampling method.

- μ has normal prior distribution, and its posterior distribution is normal also, $\mu \mid \cdot \sim N$;
- κ must be positive, and we suppose its prior distribution is normal, and then the posterior distribution is normal also. We use truncated normal distribution to make sure the sampling value is positive, $\kappa \mid \cdot \sim TN$; θ is similar to κ , and its posterior distribution is truncated normal also, $\theta \mid \cdot \sim TN$;
- ρ and σ_v is completed respectively, but we can use the method put forward by Jacquier, Polson and Rossi (2004) to sample them jointly, $\rho,\sigma_v^2 \mid \cdot \sim NIGa$, where $NIGa$ is Normal-Inverse Gamma posterior distribution;
- γ has normal prior distribution, and the posterior distribution is also normal, $\gamma \mid \cdot \sim N$;
- σ_J^2 has inverse Gamma prior distribution, and the posterior distribution is also inverse Gamma: $\sigma_J^2 \mid \cdot \sim IGa$.

3.5 Sampling latent variables

There are three latent variable in the model, $\{V_t\}_{t=1}^T$, $\{J_t\}_{t=2}^T$ and $\{G_t\}_{t=2}^T$. For latent variable series J_t at time t, it has standard distribution form. So we can use Gibbs sampling method to sample J_t. However, latent variable $\{V_t\}_{t=1}^T$ and $\{G_t\}_{t=2}^T$ do not have standard distribution form, we can use slice sampling method to sample, and calculate posterior mean and standard deviation then.

What should be emphasized is that the goal function for parameter v is complicated, we can use Metropolis-Hasting algorithm to sample, but we can use slice sampling method also. The later method is easy to implement. In SVNIG model, posterior distribution of parameter v is inverse Gamma distribution, we can use Gibbs sampling approach.

4. Data and empirical results

4.1 Simple statistics

In this section, we use Hushen 300 (HS300) index and Hong Kong Hang Seng index (HSI) to deal with the empirical work. The data is from CSMAR database. HS300 index is from April 8, 2005 to April 26, 2011, 1470 observation totally; the time period for HSI is same but with 1499 observation totally. In Fig 1, the level and return time series for both indices are given.

Based on Fig 1, we know A-share stock market surge and down sharply during the period we considered. On the first day the Hushen 300 index published the close price is 1003.5 point, and reached the top 5891.72 point on October 16, 2007; and one years later, on November 4, 2008 about 4284 points are lost and reached 1606.73 point. From Fig 1, we know, before and after 2008, the bull and bear transmission period the market is volatile. The situation is similar for HSI index.

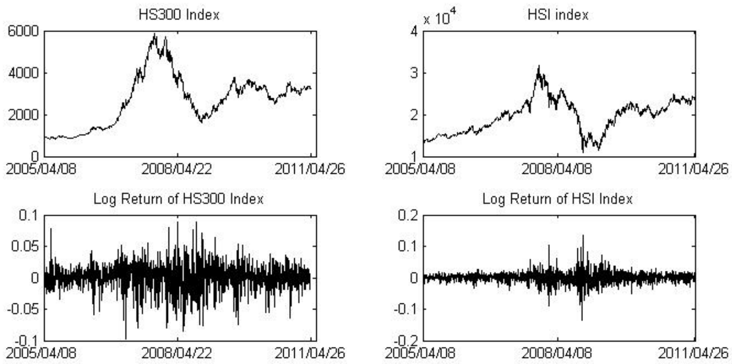

Fig 1. HS300 Index and HSI Index

In Table 1, some simple statistics for both indices are given. In these statistics, the J-B test is 372.0226 and 4339.4761 respectively, which tell us the distributions of the two return series are not normal. During the time period we considered, HS3000 index is left skewed and HSI right skewed. The kurtosis for the two indices is 5.3057 and 11.3369 respectively, which show the fat-tail and leptokurtosis properties.

Table 1. Simple statistics for return processes of HS300 and HSI

	Mean	Std	Min	Max	Skewness	Kurtosis	J-B statistics
HS300	0.0008	0.0206	-0.0970	0.0893	-0.4363	5.3057	372.0226
HSI	0.0004	0.0183	-0.1358	0.1340	0.0699	11.3369	4339.4761

4.2 Estimated parameter results

Table 2. Empirical results for the 4 models

	HS300		HIS	
	HS300-SVVG	HS300-SVNIG	HSI-SVVG	HSI-SVNIG
μ	0.4864	0.4465	0.2231	0.2023
	(-0.1114)	(-0.1067)	(-0.0724)	(-0.0701)
κ	2.1275	2.0327	2.9364	2.7127
	(-1.0125)	(-0.9375)	(-1.1824)	(-1.0309)
θ	0.1132	0.1162	0.0716	0.0822
	(-0.036)	(-0.0163)	(-0.0357)	(-0.0187)
ρ	-0.0851	-0.1045	-0.5742	-0.5653
	(-0.1458)	(-0.1655)	(-0.0814)	(-0.0849)
∅v	0.4200	0.3931	0.5155	0.4998
	(-0.0937)	(-0.109)	(-0.0683)	(-0.0622)
γ	-0.2309	-0.5902	-0.0975	-0.2357
	(-0.1108)	(-0.2434)	(-0.091)	(-0.0500)
σJ	0.1608	0.3738	0.1232	0.3464
	(-0.0139)	(-0.0387)	(-0.0216)	(-0.0432)
υ	0.1470	0.3202	0.1361	0.3171
	(-0.0137)	(-0.0791)	(-0.0046)	(-0.0354)

Note：posterior standard error in the parenthesis.

Using MCMC approach to estimate the two different models, we obtain parameters' posterior mean and latent variables, which are showed in Table 2. In our MCMC simulations, we discard the first 30,000 runs as burn-in period and use the last 20,000 iterations to estimate model parameters. Specifically, we take the means and the standard deviations of the posterior samples as parameter estimates and standard errors, respectively. Using HS300 and HSI to estimate the SVVG and SVNIG model, we call the 4 models as HS300-SVVG, HS300-SVNIG, HSI-SVVG and HSI-SVNIG model, correspondingly.

Parameter ρ are not significant in HS300-SVVG and HS300-SVNIG models, which show there is no leverage in A-share stock market during the period we considered. On the contrary, leverage effect is obvious in Hong Kong market; Parameter κ is used to measure the mean-reverting velocity. Compared with A-share market, Hong Kong market has faster convergence velocity in both SVVG and SVNIG models; Parameter θ is used to measure long-term volatility. The empirical results tell us A-share market has bigger long-term volatility than Hong Kong market.

Parameter γ is negative in the 4 models, which means in both markets jumps tend to be downward. Since the absolute value of γ in HS300 index is larger than in HSI index, that means A-share tends to produce big negative jumps, compared with Hong Kong market. Parameter σ_J is used to measure the standard deviation of jump components in the 4 models, and results show that the jump component in A-share market is more volatile than Hong Kong market.

4.3 Estimated latent variables results

In Fig 2 and Fig 3, we report the estimated results of latent variables in both SVVG and SVNIG models using HS300 and HSI indices. For the convenience of comparison, we give the corresponding return series also in Fig 2 and Fig 3. We will illustrate some phenomenon existing in the jump time series based on HS300-SVVG and HSI-SVVG models, and these phenomena exist in the results of corresponding HS300-SVNIG and HSI-SVNIG models.

In HS300-SVVG model, there are only a few big jumps, and most of them are negative. In HSI-SVVG model, big jumps are also rare, but there exists relatively much positive jumps. Small jumps are massive in both two models, which shows that Lévy process could capture both types of jumps in return process.

For both indices, one phenomenon is obvious: when the volatile of return process is violent, the value of V_t is relatively high, and the corresponding jump component is small. So, when we consider the total volatility of return process, we should check both the continuous components and the jump components.

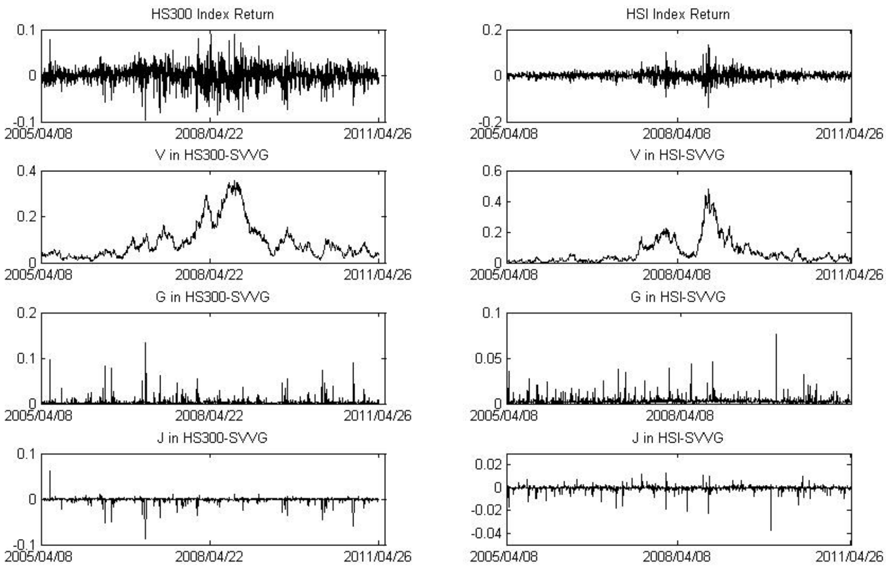

Fig 2. Latent Variables in SVVG model for HS300 and HSI Indices

Fig 3 is the estimated results of SVNIG model using two indices. Most the results we talked above hold in Fig 3. For Latent variable V_t, the concrete values for SVVG and SVNIG are different, but the difference is small. And the shapes of V_t in SVVG and SVNIG models are quite similar. This phenomenon is true for both HS300 index and HSI index. That means our two models are consistent when dealing with the same index. This is also true for latent variable J_t.

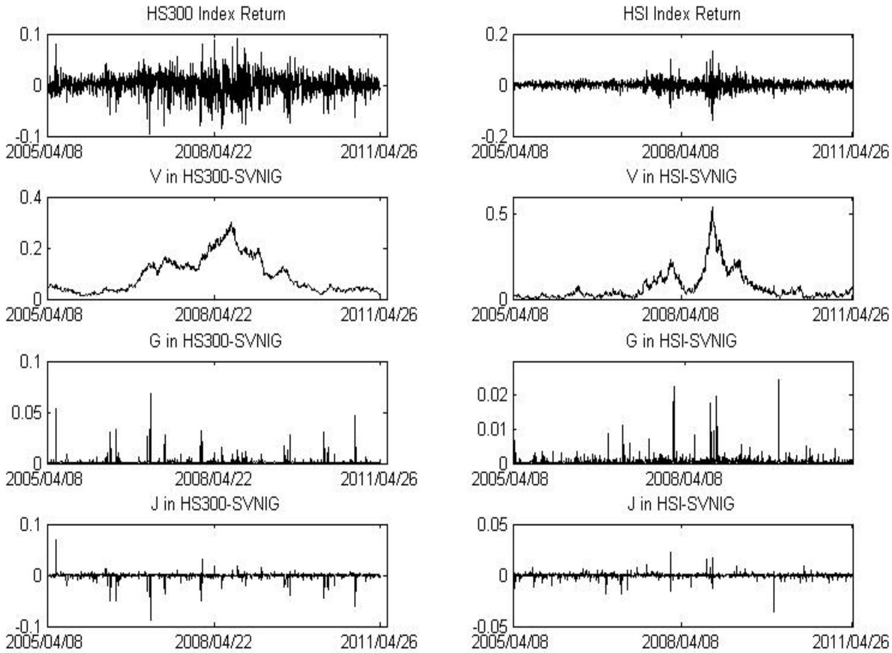

Figure 2: Latent Variables in SVNIG model for HS300 and HSI Indices

In order to check whether the model setting is appropriate, we also test the properties of the residuals. When the model setting is right, the residuals should follow standard normal distribution. Here, the Kolmogorov-Smirnov test is employed to check this property. The results are as following Table 3.

Table 3. Kolmogorov-Smirnov Test

Model	HS300-SVVG	HS300-SVNIG	HSI-SVVG	HSI-SVNIG
p-value	0.1605	0.1945	0.1107	0.2187

Based on Table 3, we know that residuals of SVNIG model are closer to standard normal distribution than corresponding SVVG model whether for HS300 index or HSI index. Therefore, compared with finite variation Lévy process, time-changed Lévy process with infinite variation behaves better.

5. Conclusion

In financial engineering, especially for asset pricing and risk management, volatility is one crucial important factor. Actually, there are at least three different volatility components in stock prices, continuous volatility, big jumps and small jumps. Traditional Poisson process could only capture big jumps but not small jumps. In this paper, under the continuous-time framework, we use time-changed Lévy process to construct one SVNIG model to capture both types of jumps. For model estimation, MCMC approach is adopted.

Using the HS300 and HSI indices, the empirical results show that besides quite a few big jumps in return process, there are a great amount of small jumps. For the HS300 index, most of the big jumps are negative. Using Kolmogorov-Smirnov test to check the residuals of SVVG and SVNIG models, we found time-changed Lévy process with infinite variation is better than SVVG model.

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